## Progression in Math

## Intent

Mathematics equips pupils with the uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem solving skills and the ability to think in abstract ways. Mathematics is important in everyday life. It is integral to all aspects of life and with this in mind we endeavor to ensure that children develop a positive and enthusiastic attitude towards mathematics that will stay with them. The National Curriculum order for mathematics describes in detail what pupils must learn in each year group. Combined with the Christ Church Primary Calculation Policy, this ensures continuity and progression and high expectations for attainment in mathematics.

It is vital that a positive attitude towards mathematics is encouraged amongst all of our pupils in order to foster confidence and achievement in a skill that is essential in our society. At Christ Church, we use the National Curriculum for Mathematics (2014) as the basis of our mathematics programme. We are committed to ensuring that all pupils achieve mastery in the key concepts of mathematics, appropriate for their age group, in order that they make genuine progress and avoid gaps in their understanding that provide barriers to learning as they move through education. Assessment for Learning, an emphasis on fluency, problem solving, the development of mathematical thinking and a rigorous approach to the development of teacher subject knowledge are essential components of the Christ Church Primary approach to this subject. We aim to provide the pupils with a mathematics curriculum and high quality teaching to produce individuals who are numerate, creative, independent, inquisitive, enquiring and confident. We also aim to provide a stimulating environment and appropriate resources so that pupils can develop their mathematical skills to the best of their ability.
Our pupils should:

- have a well-developed sense of the size of a number and where it fits into the number system
- know by heart number facts such as number bonds, multiplication tables, doubles and halves
- use what they know by heart to figure out numbers mentally
- calculate accurately and efficiently, both mentally and in writing and paper, drawing on a range of calculation strategies
- recognise when it is appropriate to use a calculator and be able to do so effectively
- make sense of number problems, including non-routine/'real' problems and identify the operations needed to solve them
- explain their methods and reasoning, using correct mathematical terms
- judge whether their answers are reasonable and have strategies for checking them where necessary
- suggest suitable units for measuring and make sensible estimates of measurements
- explain and make predictions from the numbers in graphs, diagrams, charts and tables
- develop spatial awareness and an understanding of the properties of 2d and 3dshapes


## Implementation:

1 Curriculum drivers (the Arts and Possibilities) shape our curriculum breadth in Mathematics. They are derived from an exploration of the backgrounds of our students, our beliefs about high quality education and our values. They are used to ensure we give our students appropriate and ambitious curriculum opportunities.
2 Cultural capital gives our students the vital background knowledge required to be informed and thoughtful members of our community who understand and believe in British values.
3. Curriculum breadth is shaped by our curriculum drivers, cultural capital, subject topics and our ambition for students to study the best of what has been thought and said by many generations of academics and scholars.
4. Our curriculum distinguishes between subject topics and 'threshold concepts'. Subject topics are the specific aspects of subjects that are studied.
5. Threshold concepts tie together the subject topics into meaningful schema. The same concepts are explored in a wide breadth of topics. Through this 're-visiting' of the curriculum, students return to the same concepts over and over and gradually build understanding of them. In Mathematics, these threshold concepts are; Number, Measurement, Geometry and Statistics.
6. Knowledge categories: These categories help students to relate each topic to previously studied topics and to form strong, meaningful schema. In Mathematics these knowledge categories include: Place Value, Addition and Subtraction, Multiplication and Division, Fractions, Decimals, Algebra, Statistics (interpreting tables), Measurement, Shape (properties), Shape (Position and Direction) and Ratio and proportion.
7. Cognitive science tells us that working memory is limited and that cognitive load is too high if students are rushed through content. This limits the acquisition of long-term memory. Cognitive science also tells us that in order for students to become creative thinkers, or have a greater depth of understanding they must first master the basics, which taken time.
8. Milestones: For each of the threshold concepts three Milestones, each of which includes the procedural and Knowledge categories in each subject give students a way of expressing their understanding of the threshold concepts. Milestone 1 is taught across Years 1 and 2, milestone 2 is taught across Year 3 and 4 and milestone 3 is taught across Year 5 and Year 6
9. Cognitive Domains: Within each Milestone, students gradually progress in their procedural fluency and semantic strength through three cognitive domains: basic, advancing and deep. The goal for students is to display sustained mastery at the 'advancing' stage of understanding by the end of each milestone and for the most able to have a greater depth of understanding at the 'deep' stage.

| Basic |  | Progression through the Cognitive Domains |
| :--- | :--- | :--- |
| Advancing | Deep |  |
| Acquiring knowledge. | Applying knowledge. | Reasoning with knowledge. |
| Knowledge is explicit and unconnected. | Knowledge is explicit and connected. | Knowledge is connected and tacit. <br> Relying on working memory. <br> memory to consider application. |
| Procedures processed one at a time with <br> conscious effort. | Procedures being automatic. | Relies on long-term memory, freeing working <br> memory to be inventive. |
| Understands only in the context in which the <br> materials are presented. | Sees underlying concepts between familiar <br> contexts. | Uses conceptual understanding in unfamiliar <br> situations. |
| New information does not readily stick. <br> Schemes are limited. | New information is linked to prior knowledge. <br> Schemas are strong. | Readily assimilates new information into rapidly <br> expanding schemas. |
| Struggles to search for problem solutions. <br> Relies on means-end analysis. | Combines searching for problem solutions with <br> means-end analysis. | Draws on a vast store of problem solutions. |
| Requires explicit instructions and models. | Uses models effectively. | Prefers discovery approaches to learning. |

10. Key vocabulary - move the learning from basic to deep and show progression through the milestones.

11 Pedagogical Content Knowledge and Strategies: As part of our progression model we use a different pedagogical style in each of the cognitive domains of basic, advancing and deep. This is based on the research of Sweller, Kirschner and Rosenshine who argue to direct instruction in the early stages of learning and discovery-based approaches later. We use direct instruction in the basic domain and problem-based discovery in the deep domain. This is called the reversal effect.
12 Also, as part of our progression model we (Progression in Mathematical Understanding) which shows our curriculum expectations in each cognitive domain.
13. Our curriculum design is based on evidence from cognitive science; three main principles underpin it:

- Learning is most effective with spaced repetition.
- Interleaving helps pupils to discriminate between topics and aids long-term retention.
- Retrieval of previously learned content is frequent and regular, which increases both storage and retrieval strength.

14. In addition to the three principles we also understand that learning is invisible in the short-term and that sustained mastery takes time.
15. Our content is subject specific. We make intra-curricular links to strengthen schema.
16. Continuous provision, in the form of daily routines, replaces the teaching of some aspects of the curriculum and, in other cases, provides retrieval practice for previously learned content.

| Milestone 1 <br> Key Stage 1 | Milestone 2 Lower Key Stage 2 | Milestone 3 Upper Key Stage 2 |
| :---: | :---: | :---: |
| Number and Place Value |  |  |
| - count to and across 100, forwards and backwards, beginning with 0 or 1 , or from any given number <br> - count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens <br> - given a number, identify one more and one less <br> - identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least <br> - read and write numbers from 1 to 20 in numerals and words. <br> - count in steps of 2,3 , and 5 from 0 , and in tens from any number, forward and backward <br> - recognise the place value of each digit in a two-digit number (tens, ones) <br> - identify, represent and estimate numbers using different representations, including the number line <br> - compare and order numbers from 0 up to 100; use <, > and = signs <br> - read and write numbers to at least 100 in numerals and in words <br> - use place value and number facts to solve problems. | - count from 0 in multiples of $4,8,50$ and 100 ; find 10 or 100 more or less than a given number <br> - recognise the place value of each digit in a three-digit number (hundreds, tens, ones) <br> - compare and order numbers up to 1000 <br> - identify, represent and estimate numbers using different representations <br> - read and write numbers up to 1000 in numerals and in words <br> - solve number problems and practical problems involving these ideas. <br> - count in multiples of 6, 7, 9, 25 and 1000 <br> - find 1000 more or less than a given number <br> - count backwards through zero to include negative numbers <br> - recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) <br> - order and compare numbers beyond 1000 <br> - identify, represent and estimate numbers using different representations <br> - round any number to the nearest 10,100 or 1000 <br> - solve number and practical problems that involve all of the above and with increasingly large positive numbers | - read, write, order and compare numbers to at least 1000000 and determine the value of each digit <br> - count forwards or backwards in steps of powers of 10 for any given number up to 1 000000 <br> - interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero <br> - round any number up to 1000000 to the nearest 10, 100, 1000, 10000 and 100000 <br> - solve number problems and practical problems that involve all of the above <br> - read Roman numerals to 1000 (M) and recognise years written in Roman numerals. <br> - read, write, order and compare numbers up to 10000000 and determine the value of each digit <br> - round any whole number to a required degree of accuracy <br> - use negative numbers in context, and calculate intervals across zero <br> - solve number and practical problems that involve all of the above. |

- read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.


## Addition and Subtraction

- read, write and interpret mathematical
statements involving addition (+), subtraction
$(-)$ and equals (=) signs
- represent and use number bonds and
related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20 , including zero
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as $7=-9$.
- solve problems with addition and subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction
facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete
objects, pictorial representations, and
mentally, including:
- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check
- add and subtract numbers mentally, including:
- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds
- add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
- add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- add and subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.
- perform mental calculations, including
with mixed operations and large numbers
- identify common factors, common
multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why - solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.


## Multiplication and Division

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- recall and use multiplication and division facts for the 2,5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot - solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
- recall and use multiplication and division facts for the 3,4 and 8 multiplication tables - write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods - solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
- recall multiplication and division facts for multiplication tables up to $12 \times 12$
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1 ; dividing by 1 ; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to mobjects.
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (nonprime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a oneor two-digit number using a formal written method, including long multiplication for twodigit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a onedigit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared $\left(^{2}\right)$ and cubed ( ${ }^{3}$ )
- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes - solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a twodigit whole number using the formal written method of long division, and interpret

|  |  | remainders as whole number remainders, fractions, or by rounding, as appropriate for the context <br> - divide numbers up to 4 digits by a twodigit number using the formal written method of short division where appropriate, interpreting remainders according to the context <br> - solve problems involving addition, subtraction, multiplication and division <br> - use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. |
| :---: | :---: | :---: |
| Fractions |  |  |
| - recognise, find and name a half as one of two equal parts of an object, shape or quantity <br> - recognise, find and name a quarter as one of four equal parts of an object, shape or quantity. <br> - recognise, find, name and write fractions ${ }^{1 / 3}, 1 / 4,{ }^{2} / 4,{ }^{3 / 4}$ of a length, shape, set of objects or quantity <br> - write simple fractions for example, ${ }^{1 / 2}$ of $6=3$ and recognise the equivalence of $2 / 4$, $1 / 2$. | - count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing onedigit numbers or quantities by 10 <br> - recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators <br> - recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators <br> - recognise and show, using diagrams, equivalent fractions with small denominators <br> - add and subtract fractions with the same denominator within one whole [for example, ${ }^{5} / 7+1 / 7={ }^{6} / 7$ ] <br> - compare and order unit fractions, and fractions with the same denominators <br> - solve problems that involve all of the above. <br> - recognise and show, using diagrams, families of common equivalent fractions | - compare and order fractions whose denominators are all multiples of the same number <br> - identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths <br> - recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements $>1$ as a mixed number [for example, $2 / 5+4 / 5=6$ $/ 5=1^{1 / 5}$ ] <br> - add and subtract fractions with the same denominator and denominators that are multiples of the same number <br> - multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams <br> - read and write decimal numbers as fractions [for example, $0.71={ }^{71 / 100]}$ <br> - recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents |

- count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- add and subtract fractions with the same denominator
- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to $1 / 4,1 / 2,{ }^{3} / 4$
- find the effect of dividing a one- or twodigit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.
- round decimals with two decimal places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places
- recognise the per cent symbol (\%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100 , and as a decimal
- solve problems which require knowing percentage and decimal equivalents of $1 / 2,1 / 4,1 / 5,2 / 5,4 / 5$ and those fractions with a denominator of a multiple of 10 or 25 .
- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $1 / 4 \times 1 / 2=$ 1/8]
- divide proper fractions by whole numbers [for example, $1 / 3 \div 2=1 / 6$ ]
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8]

|  |  | - identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10 , 100 and 1000 giving answers up to three decimal places <br> - multiply one-digit numbers with up to two decimal places by whole numbers <br> - use written division methods in cases where the answer has up to two decimal places <br> - solve problems which require answers to be rounded to specified degrees of accuracy <br> - recall and use equivalences between simple fractions, decimals and percentages, including in different contexts. |
| :---: | :---: | :---: |
| Measurement |  |  |
| - compare, describe and solve practical problems for: <br> - lengths and heights [for example, long/short, longer/shorter, tall/short, double/half] <br> - mass/weight [for example, heavy/light, heavier than, lighter than] <br> - capacity and volume [for example, full/empty, more than, less than, half, half full, quarter] <br> - time [for example, quicker, slower, earlier, later] <br> - measure and begin to record the following: <br> - lengths and heights <br> - mass/weight <br> - capacity and volume <br> - time (hours, minutes, seconds) <br> - recognise and know the value of different denominations of coins and notes | - measure, compare, add and subtract: lengths ( $\mathrm{m} / \mathrm{cm} / \mathrm{mm}$ ); mass ( $\mathrm{kg} / \mathrm{g}$ ); volume/capacity (l/ml) <br> - measure the perimeter of simple 2-D shapes <br> - add and subtract amounts of money to give change, using both $£$ and $p$ in practical contexts <br> - tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24 -hour clocks - estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight <br> - know the number of seconds in a minute and the number of days in each month, year and leap year | - convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) <br> - understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints <br> - measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres <br> - calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres $\left(\mathrm{cm}^{2}\right)$ and square metres $\left(\mathrm{m}^{2}\right)$ and estimate the area of irregular shapes <br> - estimate volume [for example, using 1 $\mathrm{cm}^{3}$ blocks to build cuboids (including cubes)] and capacity [for example, using water] |

- sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]
- recognise and use language relating to dates, including days of the week, weeks, months and years
- tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.
- choose and use appropriate standard
units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature $\left({ }^{\circ} \mathrm{C}\right)$; capacity (litres/mI) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using
$>$, < and =
- recognise and use symbols for pounds
(£) and pence (p); combine amounts to make a particular value
- find different combinations of coins that equal the same amounts of money
- solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change
- compare and sequence intervals of time
- tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times
- know the number of minutes in an hour and the number of hours in a day.
- compare durations of events [for example to calculate the time taken by particular events or tasks].
- Convert between different units of measure [for example, kilometre to metre; hour to minute]
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting squares
- estimate, compare and calculate different measures, including money in pounds and pence
- read, write and convert time between analogue and digital 12- and 24-hour clocks - solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.
- solve problems involving converting between units of time
- use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.
- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places
- convert between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa
- recognise when it is possible to use formulae for area and volume of shapes
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres ( $\mathrm{cm}^{3}$ ) and cubic metres $\left(\mathrm{m}^{3}\right)$, and extending to other units [for example, $\mathrm{mm}^{3}$ and $\left.\mathrm{km}^{3}\right]$.
- recognise and name common 2-D and 3D shapes, including:


## Geometry - Properties of Shape

- draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D
- identify 3-D shapes, including cubes and other cuboids, from 2-D representations
- 2-D shapes [for example, rectangles (including squares), circles and triangles]
- 3-D shapes [for example, cuboids (including cubes), pyramids and spheres].
- identify and describe the properties of 2-

D shapes, including the number of sides and line symmetry in a vertical line

- identify and describe the properties of 3D shapes, including the number of edges, vertices and faces
- identify 2-D shapes on the surface of 3-D shapes, [for example, a circle on a cylinder and a triangle on a pyramid]
- compare and sort common 2-D and 3-D shapes and everyday objects.
shapes in different orientations and describe them
- recognise angles as a property of shape
or a description of a turn
- identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle
- identify horizontal and vertical lines and pairs of perpendicular and parallel lines.
- compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes
- identify acute and obtuse angles and compare and order angles up to two right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry.


## Geometry - Position and direction

- describe positions on a 2-D grid as coordinates in the first quadrant
- describe movements between positions as translations of a given unit to the left/right and up/down
- plot specified points and draw sides to complete a given polygon.
- know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles
- draw given angles, and measure them in degrees (o)
- identify: angles at a point and one whole turn (total 360)
angles at a point on a straight line and
$1 / 2$ a turn (total 180。)other multiples of 90。
- use the properties of rectangles to deduce related facts and find missing lengths and angles
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles.
- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius
- recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.
- identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.
- describe positions on the full coordinate grid (all four quadrants)

| movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anticlockwise). |  | - draw and translate simple shapes on the coordinate plane, and reflect them in the axes. |
| :---: | :---: | :---: |
| Statistics |  |  |
| - interpret and construct simple pictograms, tally charts, block diagrams and simple tables <br> - ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity <br> - ask and answer questions about totalling and comparing categorical data. | - interpret and present data using bar charts, pictograms and tables <br> - solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables. - interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs. <br> - solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs. | - solve comparison, sum and difference problems using information presented in a line graph <br> - complete, read and interpret information in tables, including timetables. <br> - interpret and construct pie charts and line graphs and use these to solve problems - calculate and interpret the mean as an average. |
| Ratio and Proportion |  |  |
|  |  | - solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts <br> - solve problems involving the calculation of percentages [for example, of measures, and such as $15 \%$ of 360 ] and the use of percentages for comparison <br> - solve problems involving similar shapes where the scale factor is known or can be found <br> - solve problems involving unequal sharing and grouping using knowledge of fractions and multiples. |
| Algebra |  |  |
|  |  | - use simple formulae <br> - generate and describe linear number sequences |



| Progression in Mathematics Vocabulary |  |  |
| :---: | :---: | :---: |
| Milestone 1 Key Stage 1 | Milestone 2 Lower Key Stage 2 | Milestone 3 Upper Key Stage 2 |
| Tier 2 vocab- Key vocabulary. |  |  |
| Compare - looking for points of similarity and points of difference as far as mathematical properties are concerned. <br> Concrete objectives - Objects that can be handled and manipulated to support understanding of the structure of a mathematical concept. <br> Convert - Changing from one quantity or measurement to another. <br> Total-1. The aggregate. Example: the total population - all in the population. <br> 2. The sum found by adding. <br> Symbol - A letter, numeral or other mark that represents a number, an operation or another mathematical idea. <br> Columns - A vertical arrangement for example, in a table the cells arranged vertically. <br> Representative - particular form in which the mathematics is presented <br> Pattern - A systematic arrangement of numbers, shapes or other elements according to a rule. | Equivalent - A numerical or algebraic expression which is the same as the original expression, but is in a different form which might be more useful as a starting point to solve a particular problem. <br> Evaluate - Find the value of a numerical or an algebraic expression. <br> Exchange - Change a number or expression for another of equal value. <br> Digit - One of the symbols of a number system most commonly the symbols $0,1,2,3,4,5,6,7,8$ and 9 . |  |

Pictorial representation - use pictures and images to represent the structure of a mathematical concept.

Equal - Symbol: =, read as 'is equal to' or 'equals'. and meaning 'having the same value as'.

## Estimate:

1. Verb: To arrive at a rough or approximate answer by calculating with suitable approximations for terms or, in measurement, by using previous experience.
2. Noun: A rough or approximate answer.

## Tier 3 subject specific vocabulary

## Number: Place Value

Unit - The value of a digit that relates to its position or place in a number.

Roman numerals - The Romans used the following capital letters to denote cardinal numbers:
I for 1; V for 5; X for 10; L for 50; C for 100; D for 500; M for 1000.
Sequence - A succession of terms formed according to a rule.

Number bond - A pair of numbers with a particular total e.g. number bonds for ten are all pairs of whole numbers with the total 10.

Number line - A line where numbers are represented by points upon it.

Decimal - Relating to the base ten. Most commonly used synonymously with decimal fractions where the number of tenths, hundredth, thousandths, etc. are represented as digits following a decimal point. The decimal point is placed at the right of the ones column. Each column after the decimal point is a decimal place.

Square number - A number that can be expressed as the product of two equal numbers. Example $36=6 \times 6$ and so 36 is a square number Cardinal number -
Minus - A name for the symbol -, representing the operation of subtraction.

Negative number-1. A number less than zero. Example: -0.25 . Where a point on a line is labelled 0 negative numbers are all those to the left of the zero on a horizontal numberline.
2. Commonly read aloud as 'minus or negative one, minus or negative two' etc. the use of the word 'negative' often used in preference to 'minus' to distinguish the numbers from operations upon them.
3. See also directed number and positive number.
Negative integer - An integer less than 0. Examples: -1, $-2,-3$ etc.

Cube number - A number that can be expressed as the product of three equal integers. Example: $27=3 \times 3 \times 3$.

## Equation - A mathematical statement showing

 that two expressions are equal. The expressions are linked with the symbol =Numeral - A symbol used to denote a number. The Roman numerals I, V, X, L, C, D and M represent the numbers one, five, ten, fifty, one hundred, five hundred and one thousand. The Arabic numerals $0,1,2,3,4,5,6,7,8$ and 9 are used in the Hindu-Arabic system giving numbers in the form that is widely used today.

Integer - Any of the positive or negative whole numbers and zero. Example: ...-2, -1, $0,+1,+2$...
The integers form an infinite set; there is no greatest or least integer.
Odd - An integer that has a remainder of 1 when divided by 2.

Even - An integer that is divisible by 2.
Operation - Common shorthand for the four arithmetic operations of addition, subtraction, multiplication and division.

Operator - A mathematical action: In the lower key stages 'half of', 'quarter of', 'fraction of', 'percentage of ' are considered as operations.

Ordinal number - A term that describes a position within an ordered set. Example: first, second, third, fourth ... twentieth etc.

Partition - 1. To separate a set into subsets.

> 2. To split a number into component parts. Example: the two-digit number 38 can be partitioned into $30+8$ or $19+19$.
3. A model of division. Example: $21 \div 7$ is treated as 'how many sevens in 21?'
Place value - The value of a digit that relates to its position or place in a number.
Example: in 1482 the digits represent 1 thousand, 4 hundreds, 8 tens and 2 ones respectively; in 12.34 the digits represent 1 ten, 2 ones, 3 tenths and 4 hundredths respectively.
Place holder - In decimal notation, the zero numeral is used as a place holder to denote the absence of a particular power of 10 .
Example: The number 105.07 is a shorthand for
$1 \times 102+0 \times 101+5 \times 100+0 \times 10-1+7 \times$ 10-2.
Prime - The factors of a number that are prime. Example: 2 and 3 are the prime factors of $12(12=2 \times 2 \times 3)$. See also factor.

Decomposition - See subtraction by decomposition.

Digit - One of the symbols of a number system most commonly the symbols $0,1,2,3,4,5,6$, 7,8 and 9 . Examples: the number 29 is a $2-$ digit number; there are three digits in 2.95 . The position or place of a digit in a number conveys its value.

Addend - A number to be added to another
Addition - Addition is the inverse operation to subtraction

Subtract - Carry out the process of subtraction
Subtraction - The inverse operation to addition.

Subtrahend - A number to be subtracted from another.
Sum - The result of one or more additions.
Take away - 1. Subtraction as reduction / 2.
Remove a number of items from a set.
Plus - A name for the symbol +, representing the operation of addition.
Near double - See double.
Double - To multiply by 2. Example: Double 13 is $(13 \times 2)=26$.
Exchange - Change a number or expression for another of equal value.
Difference - The numerical difference between two numbers or sets of objects and is found by comparing the quantity of one set of objects with another.
Columnar addition or subtraction - A formal method of setting out an addition or a subtraction in ordered columns with each column representing a decimal place value and ordered from right to left in increasing powers of 10.

## Multiplication and Division

Commutative - A binary operation $*$ on a set S is commutative if $a * b=b * a$ for all $a$ and $b \in S$. Addition and multiplication of real numbers are commutative where $a+b=b+a$ and $a \times b=b \times a$ for all real numbers $a$ and $b$. It follows that, for example, $2+3=3+2$ and $2 \times 3=3 \times 2$.
Subtraction and division are not commutative since, as counter examples, $2-3 \neq 3-2$ and $2 \div 3$ $\neq 3 \div 2$.

Remainder - In the context of division requiring a whole number answer (quotient), the amount

Common factor - A number which is a factor of two or more other numbers, for example 3 is a common factor of the numbers 9 and 30
This can be generalised for algebraic expressions:
for example
$(x-1)$ is a common factor of $(x-1) 2$ and $(x-1)(x+$ $3)$.
Common multiple - An integer which is a multiple of a given set of integers, e.g. 24 is a common multiple of $2,3,4,6,8$ and 12 .
Long division -

Cube number - A number that can be expressed as the product of three equal integers. Example: $27=$ $3 \times 3 \times 3$. Consequently, 27 is a cube number; it . It is the cube of 3 or 3 cubed.
Multiplicative reasoning - Multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts.
Prime number - A whole number greater than 1 that has exactly two factors, itself and 1.
remaining after the operation. Example: 29 divided by $7=4$ remainder 1 .

Repeated addition - The process of repeatedly adding the same number or amount. One model for multiplication. Example $5+5+5+5=5 \times 4$.

Repeated subtraction - The process of repeatedly subtracting the same number or amount. One model for division. Example 35-5-5-5-5-5-5$5=0$ so $35 \div 5=7$ remainder 0 .

Multiple - For any integers $a$ and $b$, $a$ is a multiple of $b$ if $a$ third integer $c$ exists so that $a=b c$

Multiplicand - A number to be multiplied by another.
e.g. in $5 \times 3,5$ is the multiplicand as it is the number to be multiplied by 3 .
Multiplication - Multiplication (often denoted by the symbol " $x$ ") is the mathematical operation of scaling one number by another. It is one of the four binary operations in arithmetic (the others being addition, subtraction and division).

Inverse operations - Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. 5+6-6 = 5. Multiplication and division are inverse operations e.g. $6 \times 10 \div 10=6$.

Least common multiple - The common multiple of two or more numbers, which has the least value.

Multiply - Carry out the process of multiplication.


Answer 288
Short division


Answer: $45^{1 / 11}$
Short multiplication
$342 \times 7$ becames
342


Long multiplication
$124 \times 26$ becomes
$\begin{array}{ll}1 & 2 \\ 1 & 2\end{array}$ * 26

| 2 | 4 | 8 | 0 |
| :--- | :--- | :--- | :--- |
|  | 2 | 2 |  |


| 3 | 2 | 2 |
| :--- | :--- | :--- |

Answer: 3224
Highest common factor - The common factor of two or more numbers which has the highest value. Example: 16 has factors $1,2,4,8,16.24$ has factors $1,2,3,4,6,8,12,24$.
Prime factor - The factors of a number that are prime. Example: 2 and 3 are the prime factors of 12 ( $12=2 \times 2 \times 3$ ). See also factor.

Examples: 2 (factors 2, 1), 3 (factors 3, 1). 51 is not prime (factors 51, 17, 3, 1).

Factor - When a number, or polynomial in algebra, can be expressed as the product of two numbers or polynomials, these are factors of the first.
Examples: 1, 2, 3, 4, 6 and 12 are all factors of 12 because $12=1 \times 12=2 \times 6=3 \times 4$ :
$(x-1)$ and $(x+4)$ are factors of $(x 2+3 x-4)$ because $(x-1)(x+4)=(x 2+3 x-4)$
Dividend - In division, the number that is divided.
E.g. in $15 \div 3,15$ is the dividend

See also Addend, subtrahend and multiplicand.
Divisibility - The property of being divisible by a given number. Example: A test of divisibility by 9 checks if a number can be divided by 9 with no remainder.

Divisible - A whole number is divisible by another if there is no remainder after division and the result is a whole number. Example: 63 is divisible by 7 because $63 \div 7=9$ remainder 0 . However, 63 is not divisible by 8 because $63 \div 8=7.875$ or 7 remainder 7.

## Division -

1. An operation on numbers interpreted in a number of ways. Division can be sharing - the number to be divided is shared equally into the stated number of parts; or grouping - the number of groups of a given size is found. Division is the inverse operation to multiplication.
2. On a scale, one part. Example: Each division on a ruler might represent a millimetre.

Divisor - The number by which another is divided. Example: In the calculation $30 \div 6=5$, the divisor is 6 . In this example, 30 is the dividend and 5 is the quotient.

Double -1. To multiply by 2 . Example: Double 13 is $(13 \times 2)=26$.
2. The number or quantity that is twice another.

## Example: 26 is double 13.

Divide - To carry out the operation of division.

## Fractions

## Unit fraction - A fraction that has 1 as the

 numerator and whose denominator is a non-zero integer. Example: $1 / 2,1 / 3$.Mixed number - A whole number and a fractional part expressed as a common fraction. Example: 2 $1 / 4$ is a mixed number. Also known as a mixed fraction.
Simple fraction - A fraction where the numerator and denominator are both integers. Also known as common fraction or vulgar fraction.
Numerator - In the notation of common fractions, the number written on the top - the dividend (the part that is divided).
Denominator - In the notation of common fractions, the number written below the line i.e. the divisor.
Fraction - The result of dividing one integer by a second integer, which must be non- zero. Proper fraction - A proper fraction has a numerator that is less than its denominator So $3 / 4$ is a proper fraction, whereas $4 / 3$ is an improper fraction (i.e. not proper).
Equivalent fraction - Fractions with the same value as another. For example: $4 / 8,5 / 10,8 / 16$ are all equivalent fractions and all are equal to $1 / 2$.

Common fraction - A fraction where the numerator and denominator are both integers. Also known as simple or vulgar fraction.
Decimal fraction - Tenths, hundredths, thousandths etc represented by digits following a decimal point.

Mixed fraction - A whole number and a fractional part expressed as a common fraction. Example: $11 / 3$ is a mixed fraction. Also known as a mixed number.
Simplify - Reduce a fraction to its simplest form. Improper fraction - An improper fraction has a numerator that is greater than its denominator. minor divisions to represent minutes. The clock has two hands that rotate about the centre. The minute hand completes one revolution in one

## hour, whilst the hour hand completes one

 revolution in 12 hours. Sometimes the Roman numerals XII, I, II, III, IV, V1, VII, VIII, IX, X, XI are used instead of the standard numerals used today.Digital clock - A clock that displays the time as hours and minutes passed, usually since midnight. Example: four thirty in the afternoon is displayed as 16:30.

Capacity - Capacity - the volume of a material (typically liquid or air) held in a vessel or container.
Note: the term 'volume' is used as a general measure of 3-dimensional space and cannot always be used as synonymously with capacity. e.g. the volume of a cup is the space taken up by the actual material of the cup (a metal cup melted down would have the same volume); whereas the capacity of the cup is the volume of the liquid or other substance that the cup can contain. A solid cube has a volume but no capacity.
Centimetre - Symbol: cm. A unit of linear measure equivalent to one hundredth of a metre.

Volume - A measure of three-dimensional space. Usually measured in cubic units; for example, cubic centimetres ( cm 3 ) and cubic metres (m3).

Measure - 1. The size in terms of an agreed unit. See also compound measure.
2. Measure is also used as a verb, to find the size. Metre - Symbol: m . The base unit of length in SI (Système International d'Unités).

Mile - An imperial measure of length. 1 mile $=$ 1760 yards. 5 miles is approximately 8 kilometres.

Milli - Prefix. One-thousandth.

Millimetre - Symbol: mm. One thousandth of a metre.

## Temperature

Millilitre - Symbol: ml. One thousandth of a litre. Kilo - Prefix denoting one thousand

Kilogram - Symbol: kg. The base unit of mass in the SI (Système International d'Unités). 1kg. = 1000 g .

Kilometre - Symbol: km. A unit of length in the SI (Système International d'Unités). The base unit of length in the system is the metre. $1 \mathrm{~km} .=1000 \mathrm{~m}$.

Litre - Symbol: I. A metric unit used for measuring volume or capacity. A litre is equivalent to 1000
cm3.
Time - 1. Progress from past, to present and to future
2. Time of day, in hours, minutes and seconds; clocks and associated vocabulary
3. Duration and associated vocabulary
4. Calendar time in days, weeks, months, years
5. Associated vocabulary such as later, earlier, sooner, when, interval of time, clock today, yesterday, tomorrow, days of the week, the 12 months of a year, morning, a.m., afternoon, p.m., noon, etc.
Weight - In everyday English weight is often confused with mass. In mathematics, and physics, the weight of a body is the force exerted on the body by the gravity of the earth, or any other gravitational body.

Second - 1. A unit of time. One-sixtieth of a minute.
2. Ordinal number as in 'first, second, third, fourth ...'.

Length - The extent of a line segment between two points.
Length is independent of the orientation of the line segment
Pound - Symbol $£$. A unit of money. $£ 1.00=100$
pence.
$£ 1$ is commonly called a pound.
Symbol: lb. An imperial unit of mass. In the imperial system,
$14 \mathrm{lb}=1$ stone. 1 lb is approximately 455 grams. 1 kilogram is approximately 2.2 lb .
Mass - A characteristic of a body, relating to the amount of matter within it. Mass differs from weight, the force with which a body is attracted towards the earth's centre. Whereas, under certain conditions, a body can become weightless, mass is constant. In a constant gravitational field weight is proportional to mass.
Degree - The most common unit of measurement for angle.
One whole turn is equal to 360 degrees, written 360o
See angle

Angle - An angle is a measure of rotation and is often shown as the amount of rotation required to turn one line segment onto another where the two line segments meet at a point (insert diagram).
Angle to a point - The complete angle all the way around a point is $360^{\circ}$.
Cone - A cone is a 3-dimensionsl shape consisting of a circular base, a vertex in a different plane, and line segments joining all the points on the circle to the vertex.
Corner - In elementary geometry, a point where two or more lines or line segments meet.
Cube - In geometry, a three-dimensional figure with six identical, square faces. Adjoining edges and faces are at right angles.

## Geometry - properties of shape

Acute Angle - An angle between 0 degrees and 90 degrees.
Obtuse Angle - An angle greater than 90 degrees but less than 180 degrees.
Right Angle - One quarter of a complete turn. An angle of 90 degrees.
Net - A plane figure composed of polygons which by folding and joining can form a polyhedron.

Circumference - The distance around a circle (its perimeter).
Diameter - Any of the chords of a circle or sphere that pass through the centre.

Cuboid - A three-dimensional figure with six rectangular faces.
Trapezium - A quadrilateral with exactly one pair of sides parallel.
Triangle - A polygon with three sides. Adjective: triangular, having the form of a triangle.
Cylinder - A three-dimensional object whose uniform cross-section is a circle.
2D, 3D - Short for 2-dimensional and 3dimensional.
Vertex - The point at which two or more lines intersect. Plural: vertices.
Vertical - At right angles to the horizontal plane.
The up-down direction on a graph or map.
Line - A set of adjacent points that has length but no width.
Pentagon - A polygon with five sides and five interior angles.
Pyramid - A solid with a polygon as the base and one other vertex, the apex, in another plane.
Edge - A line segment, joining two vertices of a figure.
Equilateral - Of equal length - e.g. an equilateral triangle is a triangle with all 3 sides of equal length.
Surface - A set of points defining a space in two or three dimensions.
Symmetry - A plane figure has symmetry if it is invariant under a reflection or rotation i.e. if the effect of the reflection or rotation is to produce an identical-looking figure in the same position.
Centre - The middle point for example of a line or a circle.
Rhombus - A parallelogram with all sides equal. Rotation - In 2-D, a transformation of the whole plane which turns about a fixed point, the centre of rotation.
Quadrant - One of the four regions into which a plane is divided by the $x$ and $y$ axes in the Cartesian coordinate system.


A net of a cube
Interior Angle - At a vertex of a polygon, the angle that lies within the polygon.
Prism - A solid bounded by two congruent polygons that are parallel (the bases) and parallelograms (lateral faces) formed by joining the corresponding vertices of the polygons.
Area - A measure of the size of any plane surface.
Area is usually measured in square units e.g.
Parallel - Parallel lines, curves and planes never meet however far they are produced or extended. Always equidistant.
Parallelogram - A quadrilateral whose opposite sides are parallel and consequently equal in length. Perimeter - The length of the boundary of a closed figure.
Perpendicular - A line or plane that is at right angles to another line or plane.

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Quadrilateral - A polygon with four sides.
Isosceles Triangle - A triangle in which two sides
have the same length and consequently two
angles are equal.
Radius - In relation to a circle, the distance from
the centre to any point on the circle.
Side - A line segment that forms part of the
boundary of a figure. Also edge.
Scale Factor - For two similar geometric figures,
the ratio of corresponding edge lengths.
Rectangle - A parallelogram with an interior angle
of 90'. Opposite sides are equal.
Scalene Triangle - A triangle with no two sides
equal and consequently no two angles equal.
Polygon - A closed plane figure bounded by
straight lines.
Sphere - A closed surface, in three-dimensional
space, consisting of all the points that are a given
distance from a fixed point, the centre.
Polyhedron-Plural: polyhedra. A closed solid
figure bounded by surfaces (faces) that are
polygonal.
Hexagon - A polygon with six sides and six edges.
Heptagon - A polygon with seven sides and seven
edges.
Horizontal - Parallel to the horizon.
Oblong - Sometimes used to describe a non-
square rectangle - i.e. a rectangle where one
dimension is greater than the other
Face - One of the flat surfaces of a solid shape.
Octagon - A polygon with eight sides.
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Turn - In the opposite direction from the normal
direction of travel of the hands of an analogue
clock.
Rotation - In 2-D, a transformation of the whole
plane which turns about a fixed point, the centre
of rotation.
Vertical - At right angles to the horizontal plane.
The up-down direction on a graph or map.


|  |  |  |  | Formula - An equation linking sets of physical variables. <br> Equation - A mathematical statement showing that two expressions are equal. <br> Rule - Generally a procedure for carrying out a process. |
| :---: | :---: | :---: | :---: | :---: |
| Statistics |  |  |  |  |
| Bar Chart information Data - Inf consisting Table - 1 . numbers Tally - Ma usually by fifth coun through. <br> Chart - An <br> Interpret features mathema equation <br> Probabilit Pictogram informati | mat for rep <br> ion of a quan unts or measur derly arrang ers usually in rks to repres ing vertical l a horizontal <br> word for a t <br> out the key aph, or a cha model, or the <br> likelihood rmat for rep | g statistical <br> nature ts. <br> f information, and columns. ects counted; crossing the onal strike <br> graph <br> matical asoning, or a ns of an <br> ent happening. ng statistical | Line Graph - A graph in which adjacent points are joined by straight-line segments. <br> known as pie graph. A form of presentation of statistical information. Within a circle, sectors like 'slices of a pie' represent the quantities involved. The frequency or amount of each quantity is proportional to the angle at the centre of the circle. | Average - is a measure of central tendency which represents and or summarises in some way a set of data. <br> Mean - The mean (sometimes referred to as the arithmetic mean) of a set of discrete data is the sum of quantities divided by the number of quantities. <br> Median - The middle number or value when all values in a set of data are arranged in ascending order. <br> Mode - The most commonly occurring value or class with the largest frequency. |
| EYFS <br> New EYFS | opment Ma |  |  |  |

## Count objects, actions and sounds.

## Subitise.

Link the number symbol (numeral) with its cardinal number value.
Count beyond ten.
Compare numbers.
Understand the 'one more than/one less than' relationship between consecutive numbers.
Explore the composition of numbers to 10 .
Automatically recall number bonds for numbers $0-5$ and some to 10 .
Select, rotate and manipulate shapes to develop spatial reasoning skills.
Compose and decompose shapes so that children recognise a shape can have other shapes within it, just as numbers can.
Continue, copy and create repeating patterns.
Compare length, weight and capacity.

